$$CV = \frac{S}{\bar{X}} \times 100\% \qquad \text{[Equation 7-4]}$$

If values of precision (CV) are calculated for the people heights and mountain heights, using the previous values, we have 0.1/2 × 100% = 5% and 0.1/5,000 × 100% = 0.002%, respectively. Intuitively, these values seem more representative. Similarly, we can calculate the standard deviation of the radar gun as 2.45, and the CV or precision as 2.45/64.7 × 100% = 3.8%. We now must derive a concept and an equation for accuracy that combine the information gained from precision and bias, but to do this the concept of normal distribution must be introduced.

Normal Distribution

Consider a measurement method that has no bias; however, as with most measurement methods, repetitive measurements do not give the same results, and consequently the CV is greater than zero. Let us say that the precision of this measurement method is 10% (as defined by a coefficient of variation). Because there is no bias, the CV indicates that the average deviation from the truth is 10%. Can this value relate how large a single deviation may be or how frequently a deviation of 10%, 15%, or even 20% may occur? Of course, the smallest deviation that may occur is 0%, but it would be useful to know, for example, that some fraction of the deviations were less than some value. Assume that 95% of all deviations were within ± 20%. If we knew this, then we could make a statement about the precision of any given measurement. For example, if we measured an object one time, we could state that there was a 95% probability for the single measurement to lie within "± 20% of the theoretical mean of measurements." As there is no bias (the mean and the truth are equal), we can substitute "± 20% of the true value." A way to derive this useful information about the magnitude and the frequency of deviations is based upon statistics and involves the normal distribution.

Statistics is the science of describing or making inferences about a group of observations. The universe of possible observations from which the observations are drawn is first defined by the scientist as the population. Typically, the observations are made for only a subset of the population, known as a sample. Inferences for the entire population are then made by constructing a model, based on the sample, that represents the overall population. Models are used in virtually all branches of science and are simplifications of something more complex. The representations can be physical scale models, or they can be abstract. In the case of statistics, the models are mathematical abstractions and typically take the form of a mathematical equation relating two or more variables. The most widely used model in statistics incorporates two variables: $f(x)$ is the relative likelihood with which a certain value of the variable x occurs. An example would be the height of people, in which case x represents the values for the various heights that may occur, and $f(x)$ represents the relative likelihood of occurrence of those heights.

Figure 7-2 shows a frequency plot of repeated measurements of the constant concentration of a gas in a closed, unventilated space. Because the gas concentration is constant, the variability in the measurements is caused by random error in the measurement method. The shape of the plot is typical of randomly varying repeated measurements, in that it is peaked, and the peak is located in the center (i.e., the shape is symmetrical). Although several mathematical models may be used to approximately describe the shape of Figure 7-2, the model that has been developed most fully in statistics is the normal distribution. It is a mathematical equation relating *f(x)* to *x* and having two parameters, as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[\frac{1}{-2\sigma}(x-\mu)^2\right] \quad \text{[Equation 7-5]}$$

As Figure 7-2 shows, the typical shape of the normal distribution is bell-shaped or Gaussian. All normal curves have a similar shape to that shown in Figure 7-2. There are only two characteristics of the curves that vary: the width of the curve and the location of the curve along the x-axis. These characteristics are controlled by the two parameters μ (mu) and σ (sigma) in Equation 7-5, where μ is the location parameter or the mean of the distribution and can be estimated by $\bar{x}$. As μ changes, the curve moves left or right. As σ becomes smaller, the shape of the curve becomes more narrow and more peaked – σ being the true standard deviation, which can be estimated by s, as previously defined.

The Greek symbols σ and μ are used to indicate the population parameters, the model parameters or the truth, whereas the symbols $\bar{x}$ and s are used to indicate estimates of μ and σ. These estimates are typically derived from a sample of the population and can be used to con-

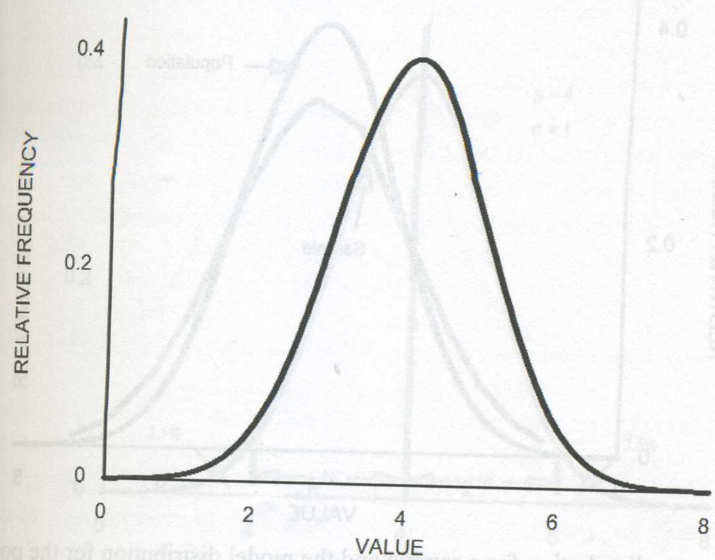

Figure 7-2. A normally distributed distribution of gas concentrations from an enclosed mixed space.

struct a normal distribution model.

Examine the two curves in Figure 7-3, which show a frequency distribution for a sample of a population and the normal distribution model based on the estimated parameters ($\bar{x}$, s) derived from that sample. The two curves have obvious differences. Why, then, is the normal distribution used as a model for something that it does not exactly replicate? First, the sample is not likely to be exactly like the population from which it is drawn. The smaller the sample size, the more likely it is that there will be differences. Second, a model allows the use of statistical techniques, shortcuts, so if the true population frequency distribution is close to normal, we are willing to accept the associated error caused by small departures from normality.

When measurements of a particular object are taken repeatedly, it is often assumed that the variability about the mean of those measurements is symmetrical or bell-shaped, that is, modeled by a normal distribution. Indeed, in measuring an object repeatedly, it seems reasonable that an individual measurement is as likely to be higher than the average as it is to be lower; hence, a symmetrical shape is expected.

There are many advantages in using the normal distribution as a model, which can be seen in their application to determining the precision, bias, and accuracy of a measurement method. These are the important features:

1. The mean and the standard deviation can be determined quite easily, and are the two parameters needed to characterize the model.

2. Regardless of the values of the mean and the standard deviation, all normal distributions have the following characteristics:

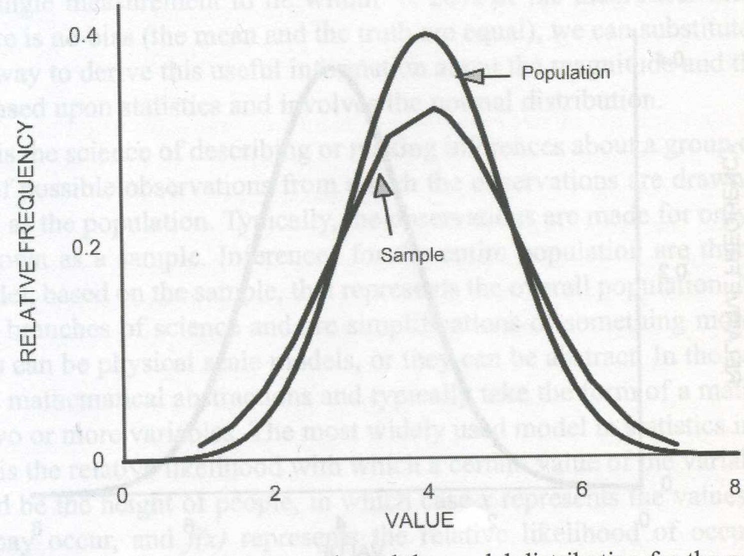

Figure 7-3. A frequency distribution for a sample and the model distribution for the population based on the mean and standard deviation estimated from the sample.

a. Plus and minus one standard deviation from the mean includes 68% of the population values; i.e., these values represent the 16th and 84th percentiles of the distribution.

b. Plus and minus two (strictly, 1.96) standard deviations from the mean includes 95% of the population values; i.e., these values represent the 2.5th and 97.5th percentiles of the distribution.

3. The mean is equal to the median is equal to the mode. All three measures of central tendency have the same value.

Many other attributes of the normal distribution are useful, particularly for testing hypotheses about a population. However, for our purposes, the above properties will suffice.

A Formal Definition of Accuracy

As stated earlier, it would be convenient if we had some way of making a statement about the precision of a method, such as "95% of all measurements lie within 20% of the mean of measurements." By using the normal distribution, we are able to construct such a statement. The precision statement is equivalent to finding the percentiles of a normal distribution. As shown in Figure 7-4, the 2.5th percentile of a normal distribution lies 1.96 or approximately two standard deviations below the mean, and the 97.5th percentile lies two standard deviations above the mean. This indicates that 5% of the population lies outside of these two percentiles, and 95% of the population lies within the two percentiles. Consequently, $\bar{x} \pm 2s$ gives the 2.5th

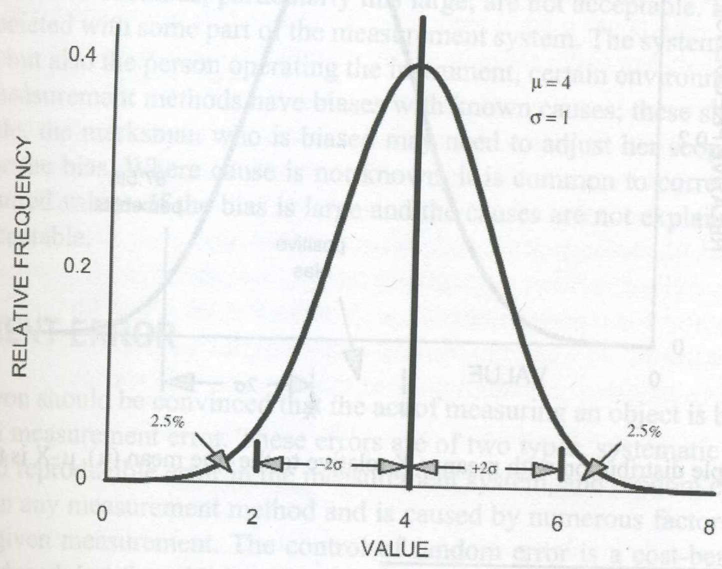

Figure 7-4. A normal distribution with $\mu = 4$ and $\sigma = 1$ showing the 2.5th and 97.5th percentiles.